THURSDAY, DECEMBER 15, 1887.

THE HORTICULTURAL SOCIETY.

THE Horticultural Society of London was founded in 1804, among the first members being Sir Joseph Banks. Its objects were "to collect every information respecting the culture and treatment of all plants and trees, as well culinary as ornamental," and "to foster and encourage every branch of horticulture, and all the arts connected with it." The Earl of Dartmouth was the first President. The Society was incorporated by Royal Charter in 1809. In 1820 the Society purchased 21 Regent Street, which was its London home for forty years. In 1822 it obtained a lease of the present Gardens at Chiswick, which have been cultivated and embellished under the Society's auspices for upwards of sixty-five years.

At the conclusion of the war in 1815 the Society began to import plants from abroad, and this country owes to its early exertions many of the beautiful camellias, azaleas, peonies, roses, and chrysanthemums which are natives of the East, and among other plants the *Wistaria* (*Glycine*) *sinensis*, a lovely creeper now quite at home in England. Indeed, one cannot take a day's ride anywhere through the country without meeting some of the beautiful introductions of the Society. Among the collectors sent out by the Society was Douglas, to whose energy the country owes *Pinus Lambertiana*, *P. insignis*, *P. ponderosa*, *P. nobilis*, *P. Douglasii*, &c.

Dr. Lindley, one of the most eminent botanists this country has ever produced, was appointed Assistant Secretary in 1822, and continued connected with the Society until his death in 1865. No account of the early days of the Society would be complete without a record of the fruits of Fortune's journeys, under its auspices, in China. Not only did he send innumerable valuable plants home, but his travels in the Chinese tea-country were the direct cause of the introduction of tea-cultivation into India.

In 1839 the Duke of Devonshire was elected President, on the death of Mr. Andrew Knight, who had been President for twenty-seven years, and to whom the Society owed much. In January 1858 the Duke of Devonshire died, and H.R.H. the Prince Consort graciously consented to succeed him. The establishment of the Society at South Kensington, under H.R.H.'s guidance and direction, is so comparatively recent an event that it is not necessary to refer to it at length. At first the prospect was promising, and had not the Prince Consort's life been cut short, the result might have been very different from what it has proved. But the money expended on the buildings and the gardens at South Kensington, from the funds of the Society, was little short of £100,000—a sum which, with the experience we now have, no one would dream of devoting to such purposes. This enormous expenditure hung like a millstone round the neck of the Society, which soon found itself unable to pay the interest on the money borrowed to meet it. The result was that, under a clause of the Society's agreement with the Commissioners of the 1851 Exhibition, the latter body resumed possession of the Gardens six years ago, and the money spent upon them by the Society was swept away at a blow.

Nevertheless the horticultural work of the Society has been carried on with undiminished energy. This surely is the proper and legitimate work of such a Society. Every departure it takes from its true functions alienates the sympathy and support of those to whom it properly looks, and to promote whose objects it exists. Since the Society has been established at South Kensington, its activity in horticultural work has been as marked as even in its most prosperous times. Many thousands of new plants, fruits, and vegetables have been submitted to the examination and the verdict of the Society's Fruit and Floral Committees, which consist of practical men, of the greatest knowledge and experience in their several departments. The value attaching to "First-Class Certificates" is shown by the care nurserymen take to record them in their catalogues. At Chiswick a long series of elaborate trials and experiments have been carried on with fruit, vegetables, and plants, whereby useful and profitable varieties have been selected and their qualities established, and inferior varieties ear-marked.

Although the Society has been unable to hold great shows owing to the loss entailed by them, it has held fortnightly shows in summer and monthly shows in winter, at which a vast number of new plants and new introductions have been seen for the first time. Such shows, though small, are often far more interesting to horticulturists than the big shows which were the fashion formerly.

It may well be asked why, if the Society can give so good an account of itself, it should be in any difficulty? The answer is that its troubles are due to its connection with South Kensington. It cannot be said that the Commissioners of the 1851 Exhibition have behaved with any conspicuous liberality to the Society. Perhaps they could not do so, as they have said that it was necessary for them to make an income out of the Royal Horticultural Gardens. But the connection with South Kensington has made it necessary for the Society to meet the views of local subscribers, who were not horticulturists; and, moreover, it has led to the Society being saddled with a charter, which prevents its expansion and adaptation to altered times and circumstances.

The views of the Council are set forth in general terms in the statement and appeal which we print elsewhere The interest in horticulture in the United Kingdom grows and spreads without check. Surely the horticulturists of the wealthiest country in the world will gladly provide the very moderate sum required for the maintenance of a Society which has done much for them, is still doing much, and has before it untold possibilities of usefulness.

BALBIN'S QUATERNIONS.

Elementos de Calculo de los Cuaterniones, &c. Por Valentin Balbin, Doctor en Ciencias, &c. (Buenos Ayres: imprenta de M. Biedma, 1887.)

ALL praise is due to the Argentine Republic for its institution of a University in which the Faculty of Sciences is endowed with a chair of the higher mathematics.

The book before us is the outcome of one of the courses of lectures which the holder of that chair, Dr.

Valentin Balbin, delivered to an audience comprising several of his colleagues. The volume, written in Spanish, has been printed at Buenos Ayres, and in size (xix. and 359 pages), and in quality of paper and print, presents a very handsome appearance.

In his preface the author informs us that in his opinion the calculus of quaternions is the best vehicle for the teaching of applied mathematics, and that therefore he has had recourse to Sir William Rowan Hamilton's beautiful invention. The author is aware of the fact that he is the first to introduce quaternions into the Spanish scientific literature, and for this reason he aims at presenting the theory from its very elements up to its higher branches of application.

In the matter of notations we are also informed that those of Hamilton and of Prof. P. G. Tait have been scrupulously adhered to, and that, in one word, the author has not found it advisable to follow those of M. Hoüel and of M. Laisant. It may not be known to everybody that these two French mathematicians have in their publications (1874, 1877, 1881) adopted a thorough reversal of Hamilton's lettering. In the place of the inventor's Greek letters, they use Roman characters (X for ρ, Y for σ, A for α, &c.); and in the place of the handy S, V, T, U, they put black-letter symbols, which are at once difficult to write, and tiring to the eyesight. These are what they call "slight alterations" or "improvements."

Again, they upset the rule about the relative appellation of the factors of a product. Our author (p. 40) states the Hamiltonian rule, and justifies it by the simple example drawn from $a + a = 2a$, where (according to everybody's ideas) the coefficient 2 is the multiplier, and a the multiplicand. According to the "innovated" rule one ought to write $a + a = a \times 2$.

The rule just named makes its influence felt more particularly in the establishment of the operator of conical rotation, and here we are sorry to find that our author falls a victim to a delusion. Instead of Hamilton's well-established $q(\ \ )q^{-1}$, he arrives at the inverse $q^{-1}(\ \ )q$ (at p. 296), and uses it under this form through several pages (up to p. 303). This comes from following M. Laisant's text, and forgetting his own rule. In M. Hoüel's opinion, "nothing is easier than to pass from one system (his own system) to the other"; nevertheless, such passage requires to be nicely managed, because by it the expression for the instantaneous axis is affected, and we might ask whether it be fair to introduce a source of confusion into a theory which in itself is difficult enough. Our author does not introduce us to the searching treatment which Prof. Tait has devoted to the question of the movement of a solid about its centre of mass ("Elementary Treatise," &c., second edition, §§ 383-400). M. Balbin's treatment of that question is very curtailed, and we might be inclined to attribute this shortness to a feeling of distrust, otherwise how could we understand his utterance, at p. 87, where he says forcibly, "Some simplifications, particularly in the physico-mathematical applications, must be made in the future as to the matter of symbols" (*se hagan*, imperative of *hacer*).

The more we consider the innovations, the more are we convinced that their proposer and his follower, publishing in 1874 and 1881, had not fully realized the extent and importance of the researches which, during many years, had been expressed in what we may term the Hamiltonian notations. In 1862 no less a Frenchman than M. Allégret set the example of following these last-named, and that precedent ought to have been adhered to. As it is, students of MM. Hoüel and Laisant will be hampered by the French notations when they approach those rich mines of information contained in such unique classics of the quaternion method as Hamilton's "Lectures" and "Elements," and the "Elementary Treatise on Quaternions," by Prof. Tait.

Let us now try to give some idea of the contents of the volume. For the English student these are all contained in the sources known to him. First "The Introduction to Quaternions," by Kelland and Tait. This work has been reproduced in its whole extent, with the exception of Chapter X., due to Prof. Tait alone. The author acknowledges in several instances (pp. 114, 120, 252) his special indebtedness to the English authors; and his translations are adequate. Perhaps, however, he knows them best through the medium of M. Laisant's reproduction of a great part of Kelland's work (with acknowledgments in the preface, tempered by the praise of the new notations).

In the second place, in the treatment of linear vector-functions and the resolution of equations involving them, which were originally given by Hamilton, there are clear indications that our author has taken his text from works where the innovated notations reign supreme; some traces of x (at pp. 183, 184, 193), for instance, are left standing in the place of ρ, and are contained concurrently with ρ in one and the same equation, in several cases; no explanation about the signification of x being given. A similar fate befell the vector ρ, at p. 138, where x is put into its place by being defined: $x = ix_1 + jx_2 + kx_3$. Under this form x is introduced into the operator ∇, which in its turn undergoes a little adaptation. But all this is not the promised adhering to Hamiltonian notations.

The solution of the vector-equation $\Sigma\alpha S\beta\rho = \gamma$ is gone partially into; but the calculation of the coefficient m of the cubic (at p. 192) contains an inexact intermediate step, and the coefficient m_1 is given with the wrong sign. Finally the solution of the proposed equation (p. 193) is incorrect, owing to the absence of the factor γ in the first term of the second member. These three inaccuracies can be traced to one of the French texts.

In the third place, curves in space, and centres of curvature of those curves, and of plane sections of surfaces, subjects exhausted by Hamilton and by Prof. Tait, have been treated by our author with the help of Dr. Graefe's little volume on Quaternions (Leipzig, 1883). We might take exception to Dr. Graefe's deduction (p. 236 of Balbin's) of Meusnier's theorem, as well as of that of the curvature of a normal section of a surface. To replace a scalar, say $S\alpha\beta$, by $\frac{1}{2}(\alpha\beta + \beta\alpha)$, in order to procure an expression of the product $\alpha\beta$ separately, seems to us to be forsaking the spirit of the method of quaternions; the expression for $S\alpha\beta$ being given, and that of $V\alpha\beta$ being deducible from other considerations, it would have been far simpler to deduce $\alpha\beta$ by forming the sum $S\alpha\beta + V\alpha\beta$ straight forward. Some reticences (p. 236), and even some inaccuracies, in the text of Dr. Graefe, have been reproduced also by M. Balbín at pp. 136, 138, 247. Dr. Graefe, like other German authors on quaternions, reproduces a great part of the "Introduction to Quaternions" by Kelland and Tait, and also some parts of the "Elementary Treatise" by Prof. Tait; but after having once pronounced the name of Hamilton, he has done all in the matter of acknowledgment, and the name of Tait is not to be found in the little volume.

We now come to the fourth class of subjects treated by our Argentine author. This comprises kinematical, statical, and dynamical questions. Here we meet with the treatment, in good form, of questions included in Hamilton's "Elements," and in the second edition (1873) of Prof. Tait's "Elementary Treatise." Of this last source of information our author seems to have only a second-hand knowledge: he reproduces verbatim the contents of § 405 of the "Elementary Treatise" (second edition), but he attributes the authorship of it to M. Laisant. Evidently, M. Laisant reproduced this § 405, which treats of Foucault's pendulum, but the origin of the treatment is to be found in the Proceedings of the Royal Society of Edinburgh of 1869, *auctore* P. G. Tait. Again, by the small-print note at p. 303 we have another indication that our author was unacquainted with the contents of the two or three last chapters in the second edition of the "Treatise." Had he known them, he could not have withheld a more special acknowledgment of results worked out by the immediate follower of Hamilton.

Prof. Tait certainly can claim to have been the first to make quaternions intelligible, not alone to ordinary students, but to advanced mathematicians—"such as have the [rare] gift of putting an entirely new physical question into symbols." But the Edinburgh Professor has particular claims to the thankfulness of students of the first-named category (the writer amongst them), for, under the plea of teaching the quaternion method, he has given them an insight into those physico-mathematical questions which are so unapproachable when obscured by the apparatus of Cartesian co-ordinates. When these questions are expressed and solved in quaternion language, they acquire a clearness and a conciseness which might well astonish their original proposers—Green, let us say, Ampère, Poinsot, even Newton, not to name living workers. We cannot be expected to enumerate the list of the questions treated; we will allude only to those in which the operator ∇ is pressed into services of such marvellous fecundity, to those in which the linear vector-functions play an eminent *rôle*, and to those in which the operator of conical rotation is such a powerful auxiliary.

The last chapter of the volume contains a painstaking record of the history of quaternions. The English reader will find much of this, and even more, in the article on "quaternions" in the "Encyclopædia Britannica." We may say that the imaginaries of algebra having done good service during the process of discovery, can be safely now banished from the principles and practice of the quaternion method—unless bi-quaternions are under treatment. In the ordinary applications of the method the extraction of the square root of the members of an equation such as $\epsilon^2 = -1$ (ϵ being a unit-vector) is looked upon as impracticable, and the reason is clearly this: the combination $\epsilon\epsilon$, represented by ϵ^2, is a symbol *sui generis* just as much as ϵ itself, and cannot be decomposed or attacked—to speak the language of chemistry—by the algebraical operation of extracting the square root of it. To assimilate a unit-vector with $\sqrt{-1}$, the square root of negative unity, is as if, in the differential calculus, one were to assimilate a derivate, $\frac{dy}{dx}$, with the symbol $\frac{0}{0}$ of indetermination. We cannot resist the temptation of helping our author to preserve a little curiosity in the history of the subject. The author records the verdict of an unnamed French mathematician, who says: "Quaternions have no sense in them, and to try to find for them a geometrical interpretation is as if one were to turn out a well-rounded phrase, and were afterwards to bethink oneself about the meaning to be put into the words. . . ." This, after all, is rivalled by the verdict of a German mathematician, who simply declared the quaternion method to be "eine Verirrung des menschlichen Geistes" (an aberration of the human intellect).

GUSTAVE PLARR.

CABLE-LAYING.

On a Surf-bound Coast; or, Cable-laying in the African Tropics. By Archer P. Crouch, B.A. Oxon. (London: Sampson Low, 1887.)

IT is somewhat remarkable that the business of making and laying submarine telegraph cables—which hitherto has been a monopoly of Great Britain, and employs large numbers of skilled workmen of all kinds, of scientific men, and of sailors—should be so little understood by people not directly connected with it. Yet the daily history of any cable-laying expedition, if faithfully written, would contain matter of engrossing interest for all readers. To secure a contract on advantageous terms requires diplomatic talent of a high order. For, although the business is a British monopoly and there is no competition with the foreigner, there is all the keener competition between the rival British companies. Further, the negotiations are almost always with Government departments, either home, colonial, or foreign, and are necessarily of a delicate character. In the history of any particular cable the preliminary diplomatic details would no doubt have by far the greatest interest for most readers, but it would be obviously indiscreet and unadvisable to publish them. In tendering for a cable against powerful competitors it is important to have as accurate a knowledge as possible of the depth of water and the nature of the bottom where the cable is to lie, in order to know exactly the lengths of the different types of cable which will have to be employed, and so to estimate the cost. In obtaining this knowledge the cable-laying companies have been the chief contributors to the science of deep-sea research, or oceanography. The contract obtained, the cable made, and the route determined on, the operation of laying has to be undertaken. When it is merely a question of laying a length of cable between two points over smooth ground, this is in most cases a very simple affair; although if the shore-ends of the cable have to be landed on exposed beaches, as is only too often the case, there is plenty of opportunity for thrilling incident and hair-breadth escape. The expedition in which Mr.